Geometric Pattern and its Geometry Part 4 – Geometry from the Mughals' land

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Abstract

In this paper, we discuss selected geometry concepts used in geometric art from the Mughal Empire. We show here how selected geometric designs from Mughal architecture were constructed and what geometry was employed there. All examples are presented as step-by-step constructions with Geometer's Sketchpad (GSP), a school geometry software. One can use GeoGebra or Cinderella. Occasionally we will mention the creation of custom tools in geometry software. However, the main emphasis will be placed on geometry and geometric pattern design.

Introduction

In November 2020, I had a lecture on geometry in Mughals' architectural decorations. It was an online lecture for mathematics teachers and students. Most of the examples were presented as live constructions. Due to time constraints, the selection of examples was limited. This was the core material for this paper. A few more examples were added to get a more complete spectrum of geometry topics used in Mughal architecture. Some summaries and commentaries from my lecture were updated and expanded. The presented here material is still far from complete. The subject can be developed into an excellent course or a book on geometric concepts in art and architecture from the Indian subcontinent.

Mughals and Mughal Empire

It is not easy to describe who the Mughals were. There is no ethnic group of Mughals. Instead of this, we have several immigrant groups coming to the Indian subcontinent from various places. Even the term Mughal in multiple periods changes its meaning. As for our purpose, it is enough to know that Mughals were all Central Asian immigrants to India: Uzbek, Chughtai, Tajik, Barlas, Kipchak, Kazakhs, Turkman, Kyrgyz, Uyghurs or Mongol. This term is also used for later immigrants from Iran and Turkey, such as the famous Qizilbash community.

It is much easier to tell what was the Mughal Empire. After Wikipedia (see [7]), we have:

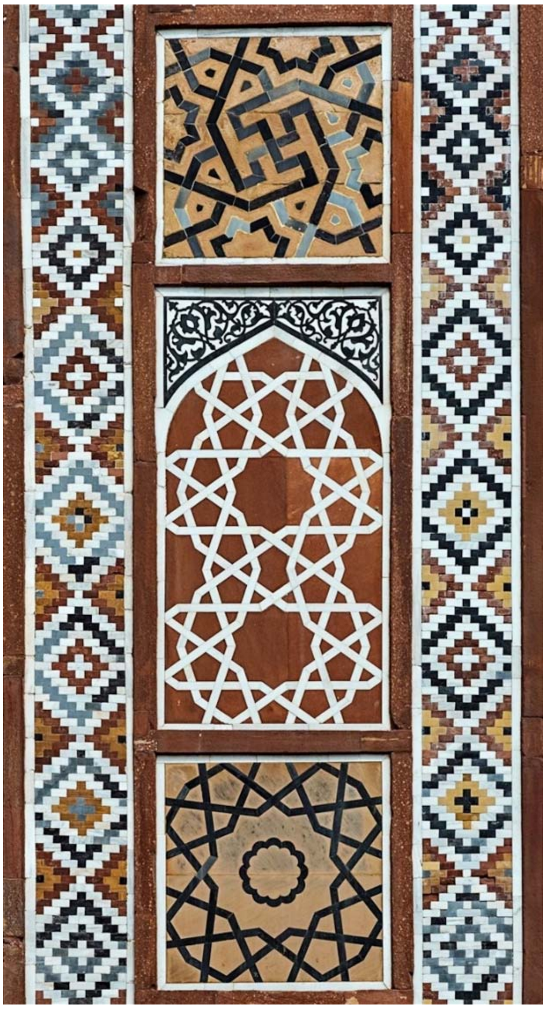
The Mughal empire [...] have been founded in 1526 by Babur, a warrior chieftain from what today is Uzbekistan, who employed aid from the neighboring Safavid and Ottoman empires, to defeat the Sultan of Delhi, Ibrahim Lodhi, in the First Battle of Panipat, and to sweep down the plains of Upper India. The Mughal imperial structure, however, is sometimes dated to 1600, to the rule of Babur's grandson, Akbar. This imperial structure lasted until 1720, until shortly after the death of the last major emperor, Aurangzeb, during whose reign the empire also achieved its maximum geographical extent. Reduced subsequently, especially during the East India Company rule in India, to the region in and around Old Delhi, the empire was formally dissolved by the British Raj after the Indian Rebellion of 1857.

The relative peace maintained by the empire during much of the 17th century was a factor in India's economic expansion. Burgeoning European presence in the Indian Ocean, and its increasing demand for

Indian raw and finished products, created still greater wealth in the Mughal courts. There was more conspicuous consumption among the Mughal elite, resulting in greater patronage of painting, literary forms, textiles, and architecture, especially during the reign of Shah Jahan. Among the Mughal UNESCO World Heritage Sites in South Asia are: Agra Fort, Fatehpur Sikri, Red Fort, Humayun's Tomb, Lahore Fort, Shalamar Gardens and the Taj Mahal, which is described as "the jewel of Muslim art in India, and one of the universally admired masterpieces of the world's heritage."

Geometry concepts in patterns from Mughal architecture

Most of the places mentioned a while ago are decorated with geometric designs displaying various geometric features. Some of them were influenced by the type of material used to make these decorations: ceramic tiles, brick, marble, and other materials. In general, we find here local materials and local colors – usually shades of sepia, ochre, black and white.



Fragment of a wall on Akbar's tomb

This image shows four geometric patterns from the Akbar tomb (see [8]).

On the sides, we have brick patterns using squares as a geometric framework. This simple type of design can be seen all over Central Asia, India, and Iran.

The top pattern is a unique combination of a swastika motif inscribed into a square and halves of four stars with D8 symmetry.

The central part occupies a large panel with decagonal stars and shapes derived from the regular decagon's geometry. This geometric pattern has mirror symmetries and local pentagonal and decagonal symmetries.

Finally, the design at the bottom also has mirror symmetries and local dodecagonal and octagonal symmetries. The star in the middle has 12 corner vertices. Around it, we have 12 identical kites forming another star. In the corners, we have quarters of stars with 8 corner vertices. A very peculiar feature of this design are pentagons that look almost like regular pentagons.

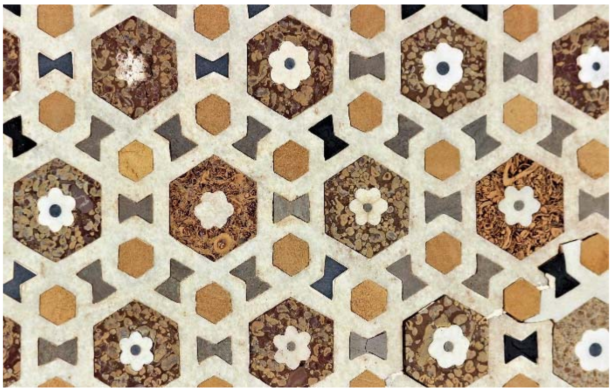
Another particular feature of the three patterns in the middle are thick lines. They are ceramic or stone narrow tiles combined in continuous linear structures. For this paper's purpose, we will replace them with thick segments in most of our constructions. This will make our work much more straightforward.

The Akbar's tomb is a geometric treasure. We can see there many more exciting designs – check the photo in [8].

This paper will discuss various geometric designs, often briefly mentioning places where one can find them. An interested reader can check Wikipedia for information about multiple monuments. The website by David Wade [9] contains an extensive collection of good quality photos. On the same website, a slideshow [10] can give us some ideas about types of designs and materials used in architectural decoration in Mughal architecture.

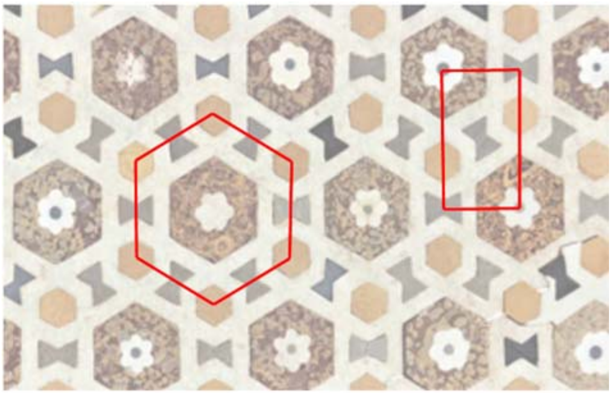
The remaining parts of this paper we will devote exclusively to analyzing geometric patterns and their construction.

Hexagonal geometry



Example 1. Hexagonal mosaic from Itimud ad-Daula tomb

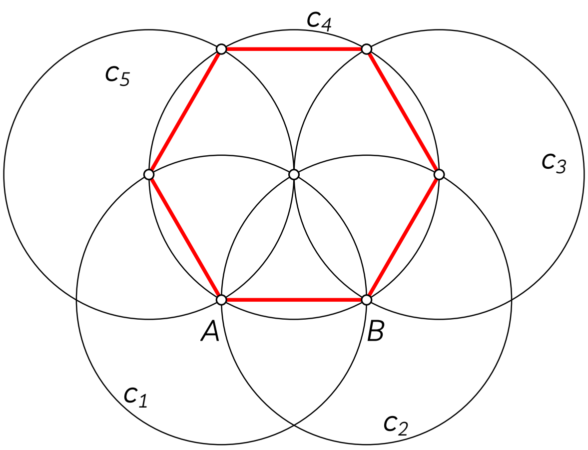
This is a very popular design. We can find it in many places all over India, Central Asia, and Iran. The one shown in the photo comes from the Tomb of Itimud ad-Daula (Mirza Ghiyas Beg's) in Agra (see [11]). A particular feature of it are regular hexagons of two different sizes. One can construct this design using a rectangular template or a hexagonal template. Here we will show both ways.

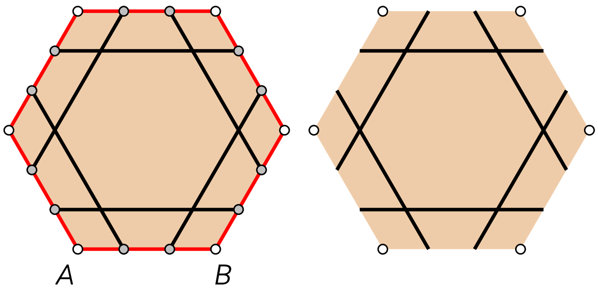


Two ways of creating this pattern

The drawing explains the two ways of reconstructing this decoration. We can use hexagonal tiles like the one shown here to produce this pattern. The same task can be obtained by using a rectangular tile with specific proportions. The second approach is more convenient for planning larger designs. But the first one is more obvious.

In the first case, one can create a custom tool in his geometry software, e.g., in Geometer's Sketchpad or GeoGebra.

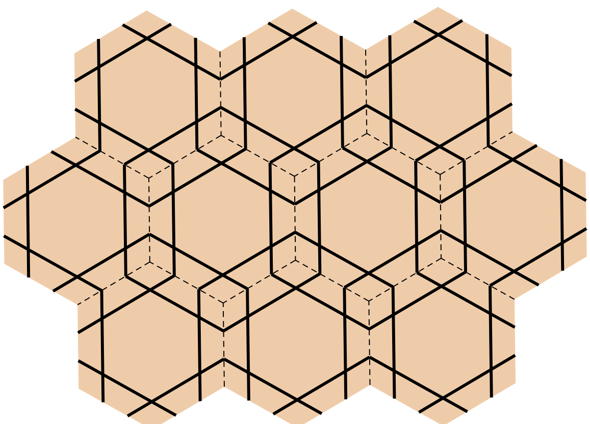




Construction of a tool for designing the pattern

(GSP) Divide each edge of the hexagon into three equal parts and connect points, as is shown here.

Select vertices of the hexagon and segments inside it and create a custom tool for drawing the tile.



Larger design created with the custom tool

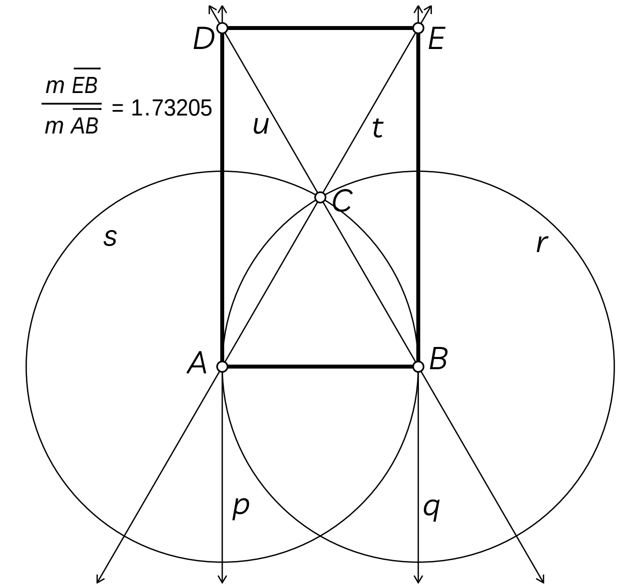
At this moment, one can figure out that such a design can be rendered in two different ways.

A sizeable mosaic panel can be created using hexagonal tiles with motifs painted on them. Dashed lines show the edges between neighboring tiles. We need one type of tiles only.

The second method uses three separate ceramic tiles: the medium-sized hexagon, the small one, and the drum-like shape. The sizes of these ceramic tiles can be calculated from the length of the edge of the large hexagon.

Both methods were used by artists in Muslim and European countries. The first one is less time consuming, and it is still often used in our bathrooms and kitchens. It seems that the second method was more prevalent in the Mughals' empire.

As we said before, the same artwork can be designed using a rectangular template.



Creation of the contour for the second method

The thick segments form a contour for the second method. Its proportion can be easily calculated. The length of AB can be set to any value we want. This is 1/(2n) of the width of the final artwork. The size of BE is twice the height of the equilateral triangle ABC.

$$BE = 2 \cdot AB \cdot \frac{\sqrt{3}}{2} = AB \cdot \sqrt{3} \approx AB \cdot 1.73205$$

In this construction, crated objects' order is as follows: A, B, s, r, C, p, q, u, t, D, E.

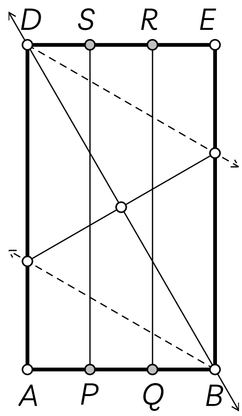
It is worth mentioning that many other hexagonal geometric patterns use the same contour or its modification. One of them we will see later.

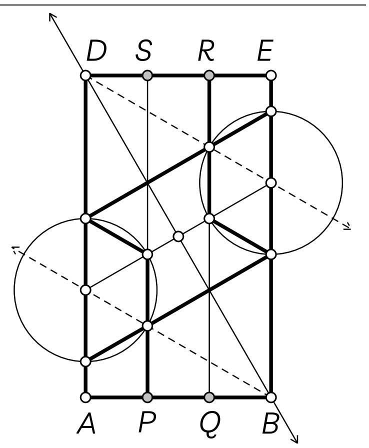
Construction of the pattern in rectangular contour

Points P, Q, and S, R were created by dividing segments AB and DE into three equal parts. Dashed lines are angle bisectors.

The thin lines form a network for drawing the pattern.

There is one crucial difference between these two methods. In the first method, we create a large design by copying and moving (translation) the hexagonal tile. In the second method, we need to create a mirror copy of the obtained design. It is shown in the next drawing.

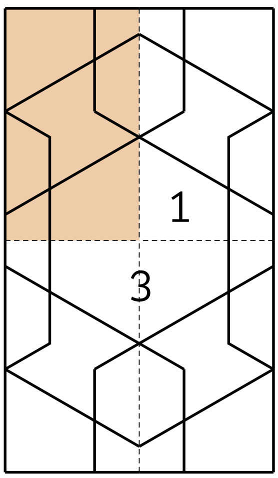


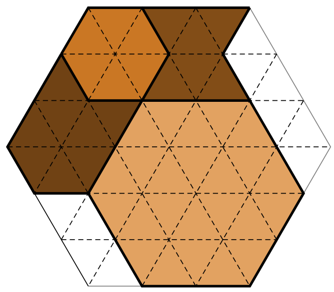


Creation of the second template

The shaded rectangle is the template created above. The three other rectangles are mirror reflections of the first rectangle. The order of creation can be like this: reflect the first rectangle about its right edge, then reflect both rectangles about their bottom edge.

This way, we created another larger template that can be used to make a large pattern by copying it and moving in the necessary direction. This reminds assembly of a floor tiling using a bunch of rectangular tiles with the design shown here (left drawing).



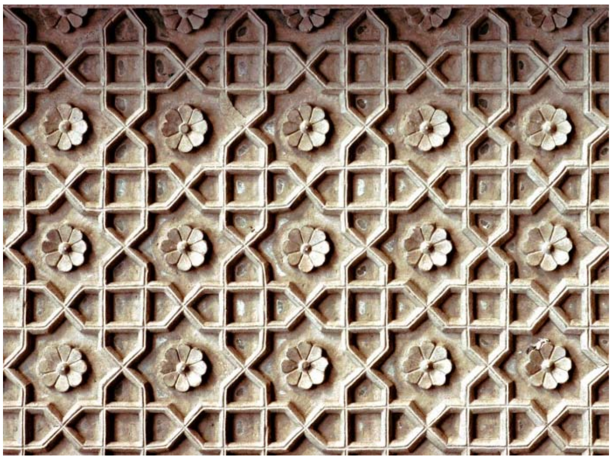


Sample shapes used in hexagonal geometric patterns

Geometric patterns in hexagonal geometry often, but not always, use various shapes derived from the geometry of an equilateral triangle and polygons derived from it (see the drawing above), and angles equal to $n{\cdot}30^\circ$.

Octagonal geometry Mughal style

We will discuss here two geometric patterns with octagonal local symmetries. The first one is straightforward. The second one has a more complex structure. Although they share the same geometric features.

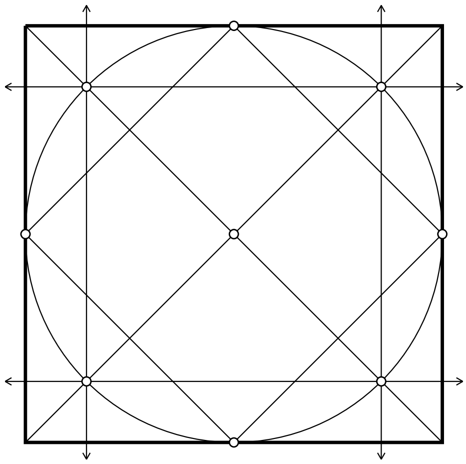


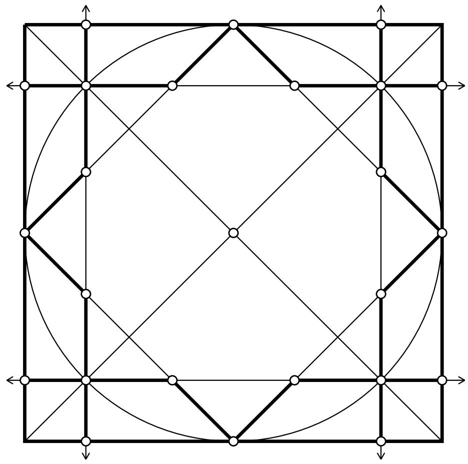
Example 2. Octagonal design from Agra fort

This design occurs probably in all regions of the Muslim world. There are many variations of it. But the key features remain the same. A pattern like this one is often called a star and cross design.

The pattern on the photograph has a significant error that is not easy to spot. The large stars with 8 corner vertices do not have all edges equal. In a typical example of this pattern in Iran and other places, the octagonal star should be very symmetric and should have all edges the same length.

Below I show the creation of a template for this pattern.





Example 3. Design from the Itimud ad-Daula tomb

Here is another octagonal design. It is more complex, and it shows a few specific shapes that are often seen on octagonal geometric patterns. We will discuss them in a moment.

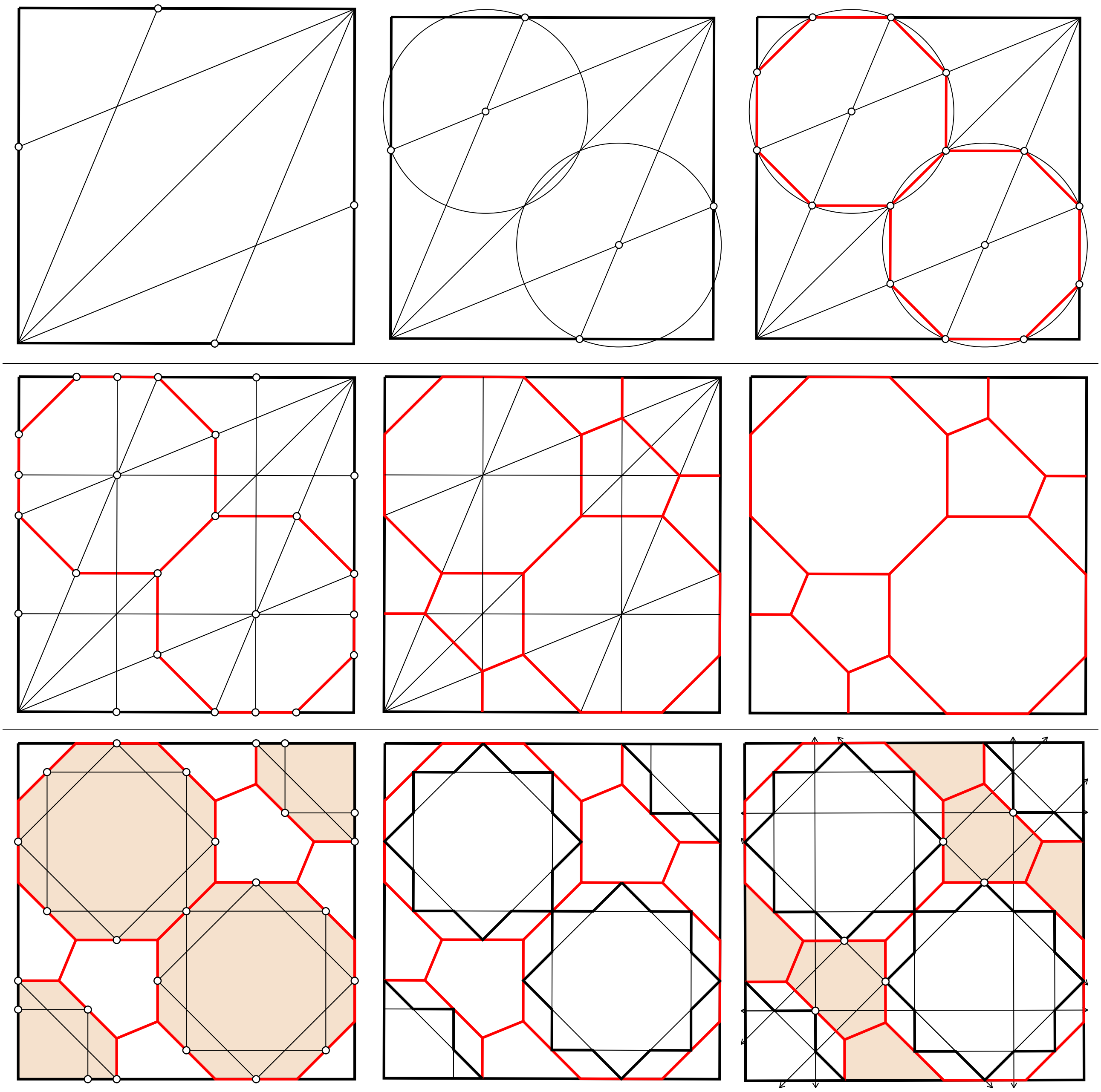
It is worth to notice how well this work was planned. The pattern is on the side of a column. If we denote by *H* its height, then its width is equal to *2H*. We have here two squares copies of the same pattern. We show this in the next figure.

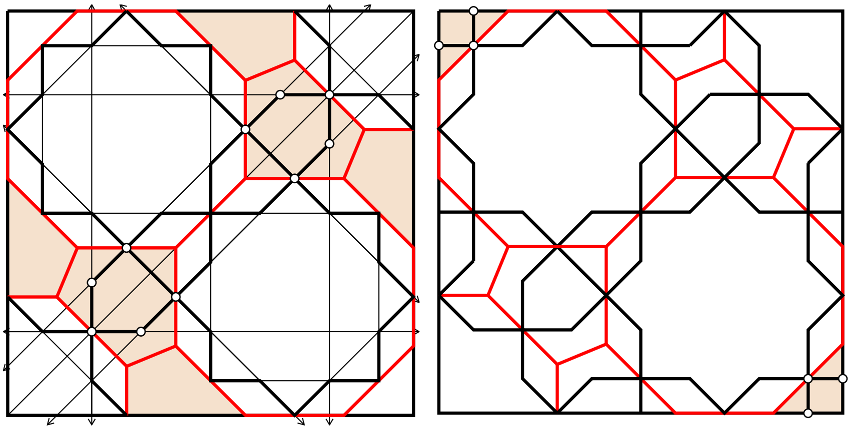
Templates for the pattern from Itimud ad-Daula tomb

This drawing shows how this geometric pattern can be understood. We have here two types of templates. The large square is a movable template. A large design can be created by translating it.

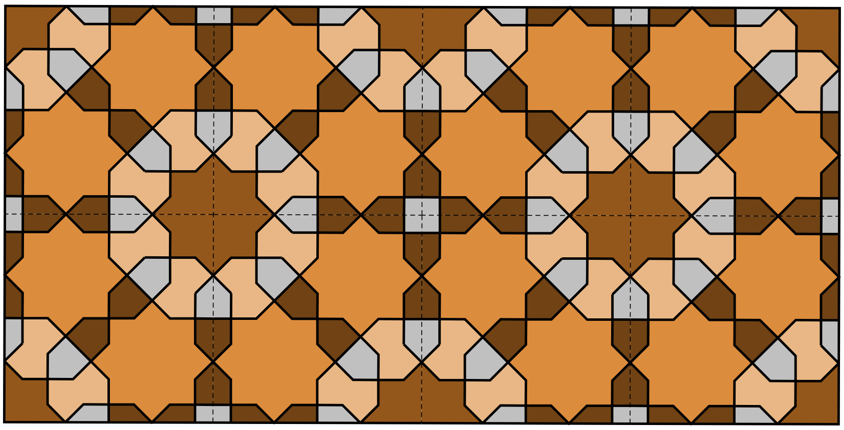
Each of the small squares can also be used as a template for a pattern, but we have to use reflections about its edges or rotations about one of its vertices.

Below we show step-by-step construction of the pattern's geometric structure (red tessellation) and the design (thick black lines).





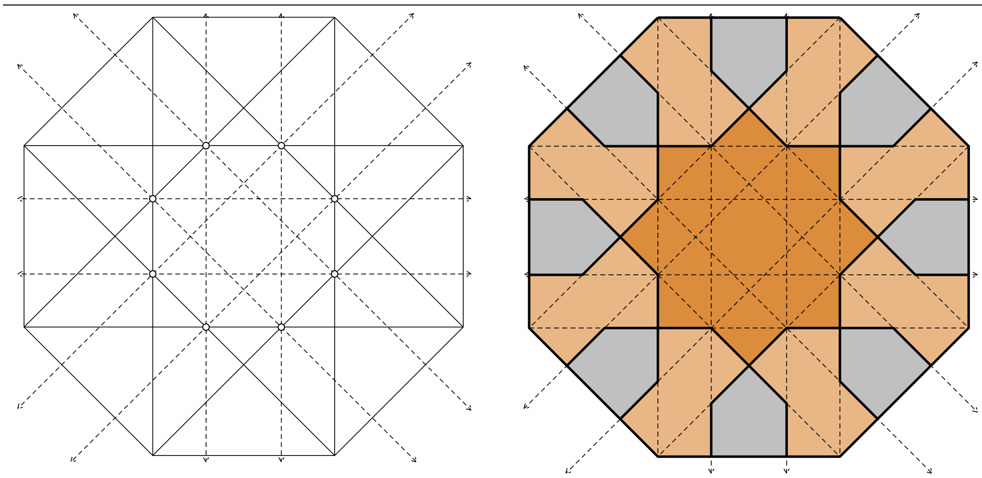
In all the images, the shaded polygons are used to emphasize places where we construct something. In all drawings, we left the necessary points only. All unnecessary construction elements are hidden.



Here we show a complete pattern from the Itimud ad-Daula tomb.

In this pattern, we have only four types of shapes: a large octagonal star, two identical pentagons (gray and dark ochre), hexagons, and a small square. In other patterns of this type, we may have more elements.

It is worth to look at the connection of elements from this design with the geometry of a regular octagon. The picture below shows this relation. In example 8, we will come back to this geometry.



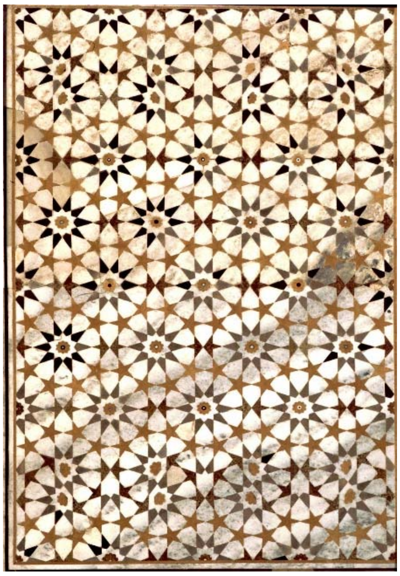
In the left drawing, continuous segments were created first. Their points of intersection, shown here, were used to develop the slant lines in the middle of the octagon.

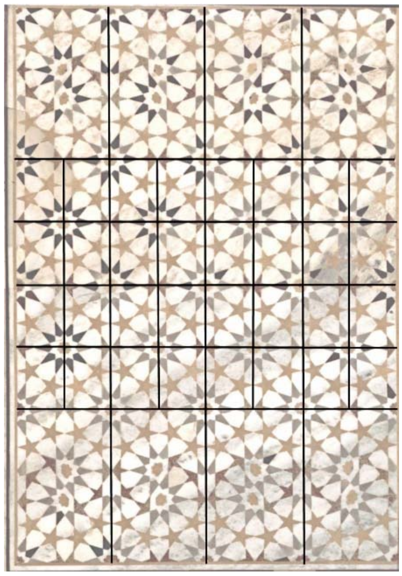
The right drawing shows how each particular element can be created. The star and the hexagons have all edges of the same size. In hexagons, we have angles 90°, 135° , and 225°. Finally, the pentagon has four equal edges, three angles 90°, and two angles 135°.

Shapes used in octagonal geometric patterns frequently use angles 90°, 135° , and 225° , and in many examples, their edges are two sizes only (compare one of the next examples). They differ significantly from these in traditional Moroccan architecture.

Decagonal geometry

Geometric patterns in decagonal geometry use completely different rules and shapes than those we discussed until now. One of the most important features is the golden ratio that occurs quite frequently in various moments.

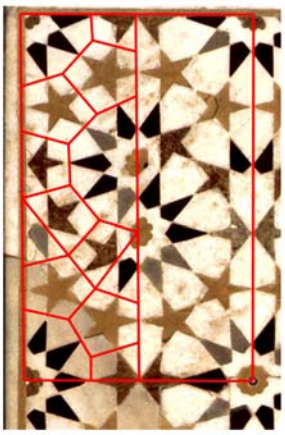


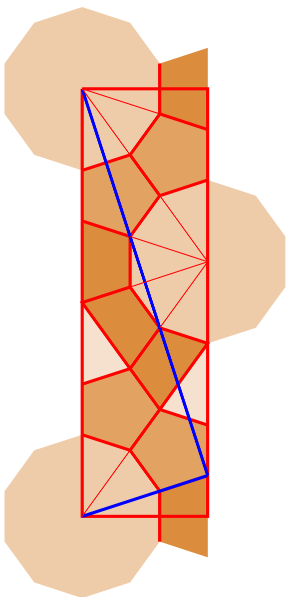


Example 4. Decagonal pattern from the Tomb of Itimud ad-Daula in Agra

Although this design looks very complicated, we can easily see that it combines two different patterns (large and small rectangles). In this paper, we will concentrate on the larger one (large rectangle). The other one was discussed in my other article (see [2]).

Another reason for choosing the larger design are the strange shapes that we can see there. They look like dragon feats with six claws.





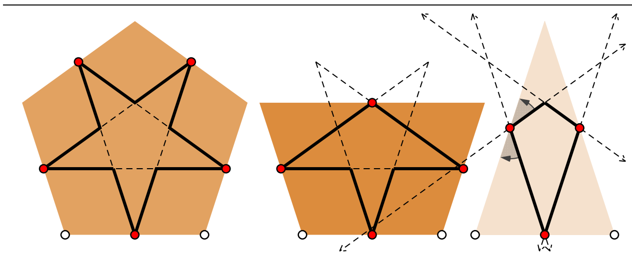
Analysis of the large pattern

These two drawings show how we can approach the reconstruction of the pattern.

Left drawing – by rough hand drawing, we can notice two things. One – the whole pattern can be split into two vertical strips, and the right one is a rotated 180 degrees copy of the left one.

This design's geometry can be constructed from regular decagons and pentagons, a trapezium, and a long triangle. The long triangle is 1/10 of the regular decagon.

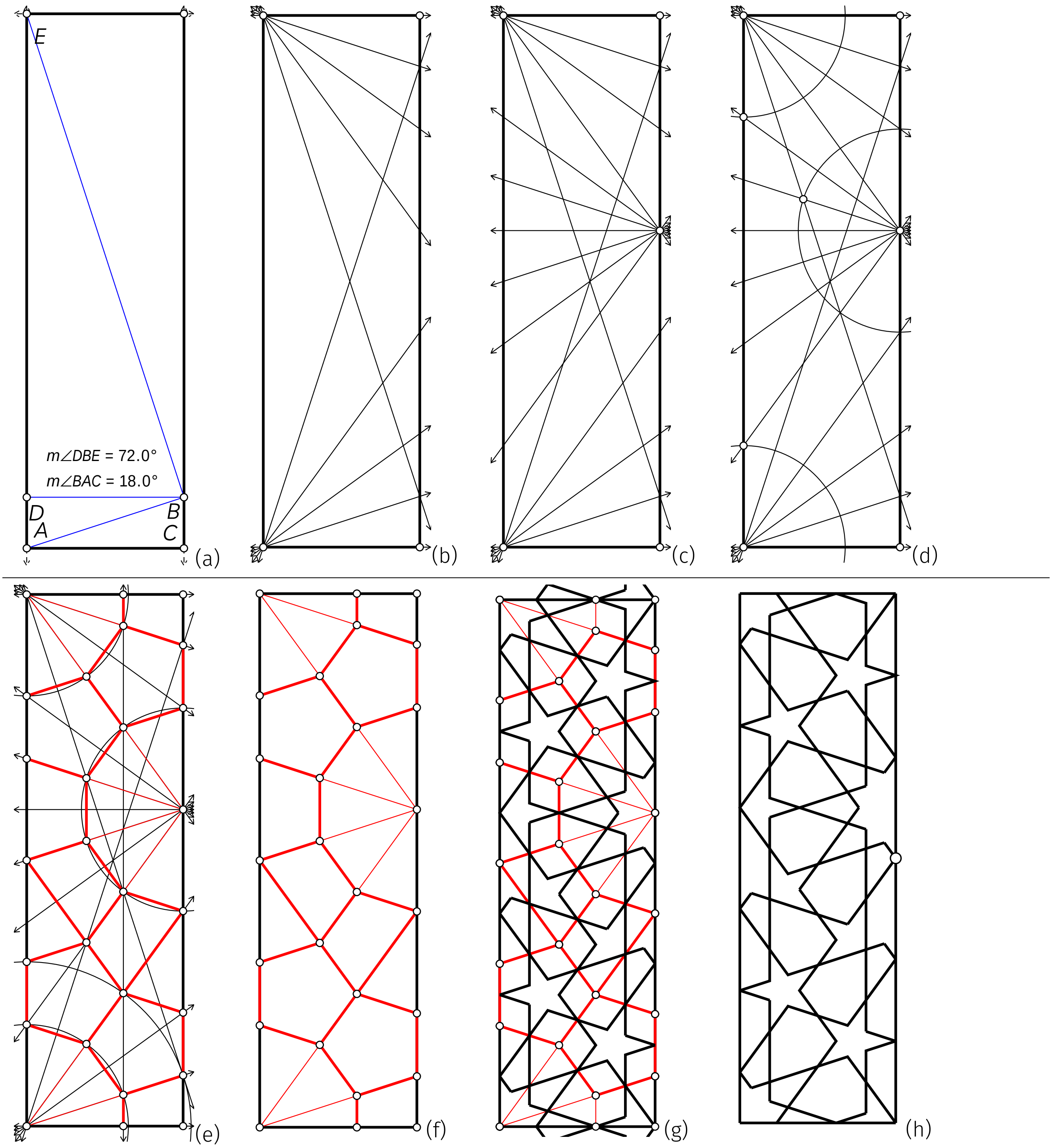
The right drawing shows the geometry of the left side of the pattern very accurately. The blue lines are hints for creating the whole contour. Below we depict all steps in this construction. We start by creating the outline, and then we construct the tessellation and, finally, the pattern.



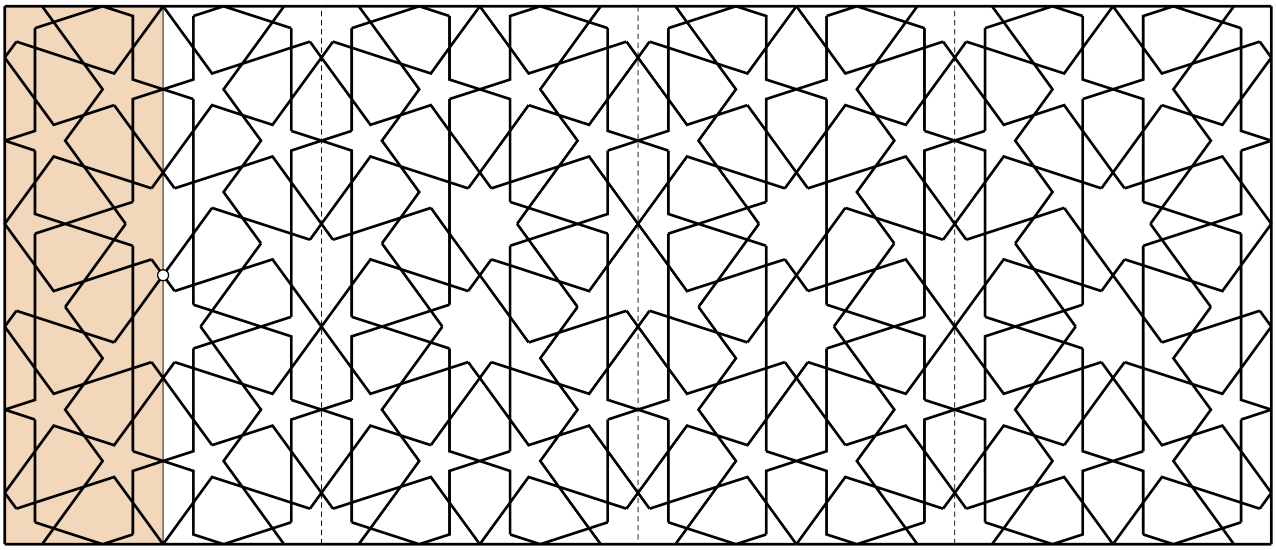
(GSP) *We will need three custom tools for this construction: the pattern for pentagon, trapezium, and long triangle.*

The red points are the midpoints of sides. The white points are the starting points for the creation of the tool.

If you draw this pattern by hand, you can construct your tessellation first and then copy the three tiles – pentagon, trapezium, and long triangle. Draw designs on these copies, and then copy them back to the tessellation.

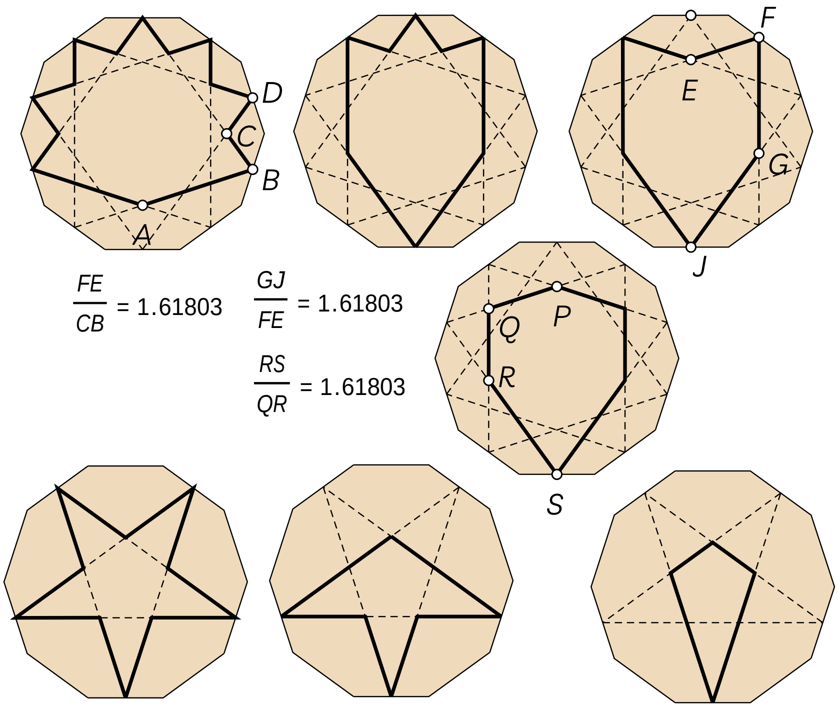


Drawing (a) shows how we create the contour for half of this design. Drawings (b) to (f) show how the tessellation was constructed. Illustration (g) shows tessellation filled with pattern. Finally, in (h), we see the left half of the pattern. The small circle on the right edge is the point where we have to rotate the design 180 degrees to produce the whole template.



Here we have a larger pattern created from 8 copies of the design from drawing (h), the shaded area. The small circle shown here marks the point where the pattern on the left was rotated 180 degrees. Dashed lines are mirror reflection lines.

The family of decagonal patterns is enormous, and it contains designs from the simplest to the most complex one can see. In India, we do not have many decagonal designs. At this moment, it is worth looking at the shapes that we used in this pattern. But we still have to remember that we are barely touching this subject. The collection of forms used in decagonal designs is large. Some of them occur very frequently, and some of them are quite rare.



Selected shapes used in this pattern

We know how shapes inside of pentagon, trapezium, and long triangle were created. Here we show a few other shapes from this pattern and their relation to a regular decagon's geometry.

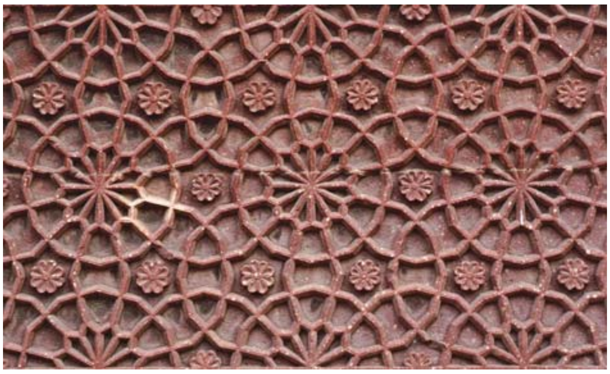
In these shapes, the edges have three lengths, and proportions of their sizes are equal to 1.61803.., which is the well-known golden ratio. All these facts can be easily proved using algebraic calculations.

Here in the bottom row, we show shapes that occur inside our tessellation tiles. In the top rows, we show shapes that occur on the peripheral parts of our tessellation tiles. For different tessellations, they may be located differently.

The existence of the golden ratio in decagonal geometric patterns is not always apparent. We can find many decagonal designs where the detection of the golden ratio is more complicated.

Dodecagonal geometry

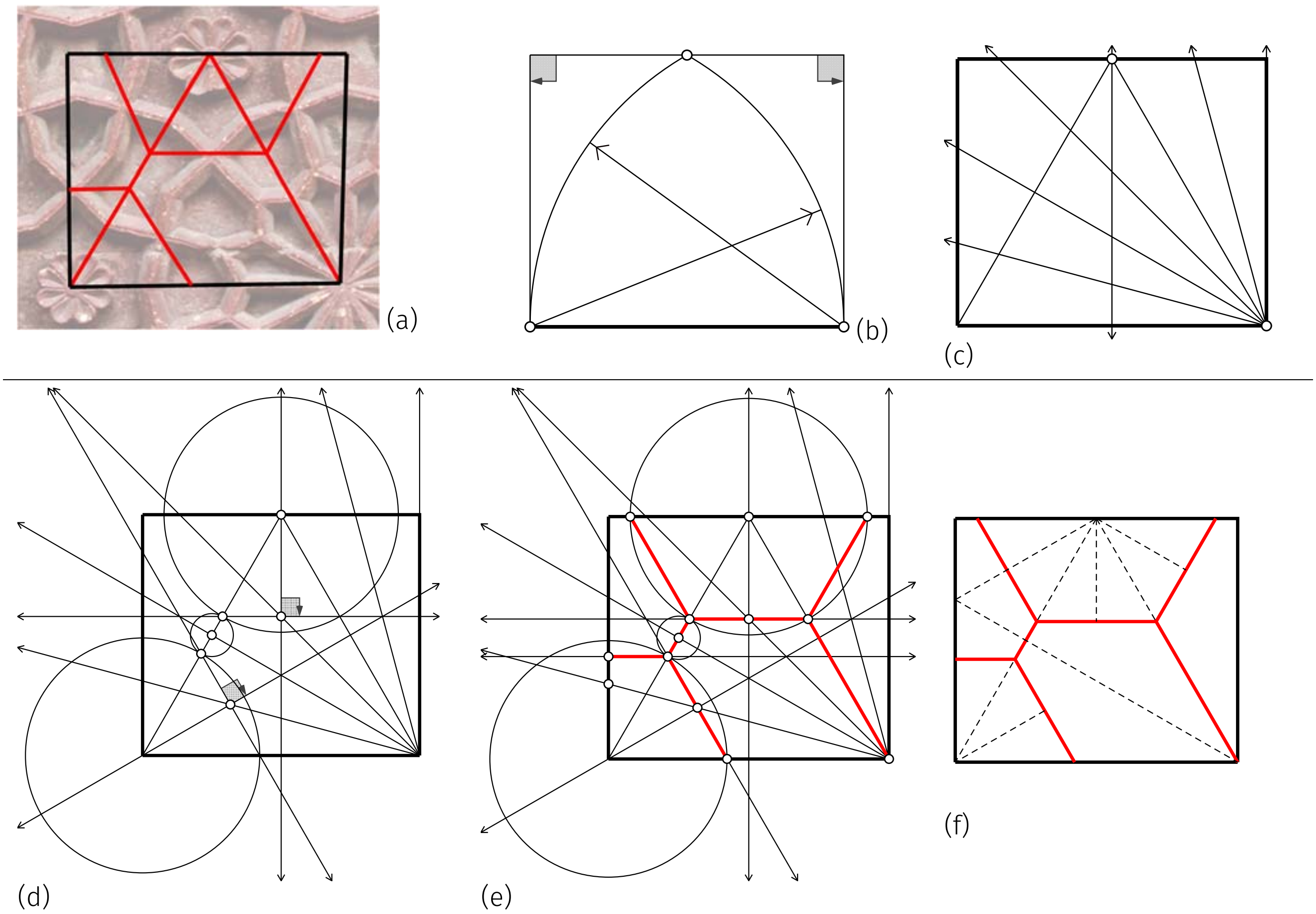
The geometry of patterns with dodecagonal local symmetries brings us into a completely different world. We will have different angles, different tessellations, and different shapes. Tessellations of these patterns are often tessellations with regular polygons, usually equilateral triangles, squares, regular hexagons, regular dodecagons, etc. Again this is a very rich world. But its geometric properties are more variable than in other groups of patterns.

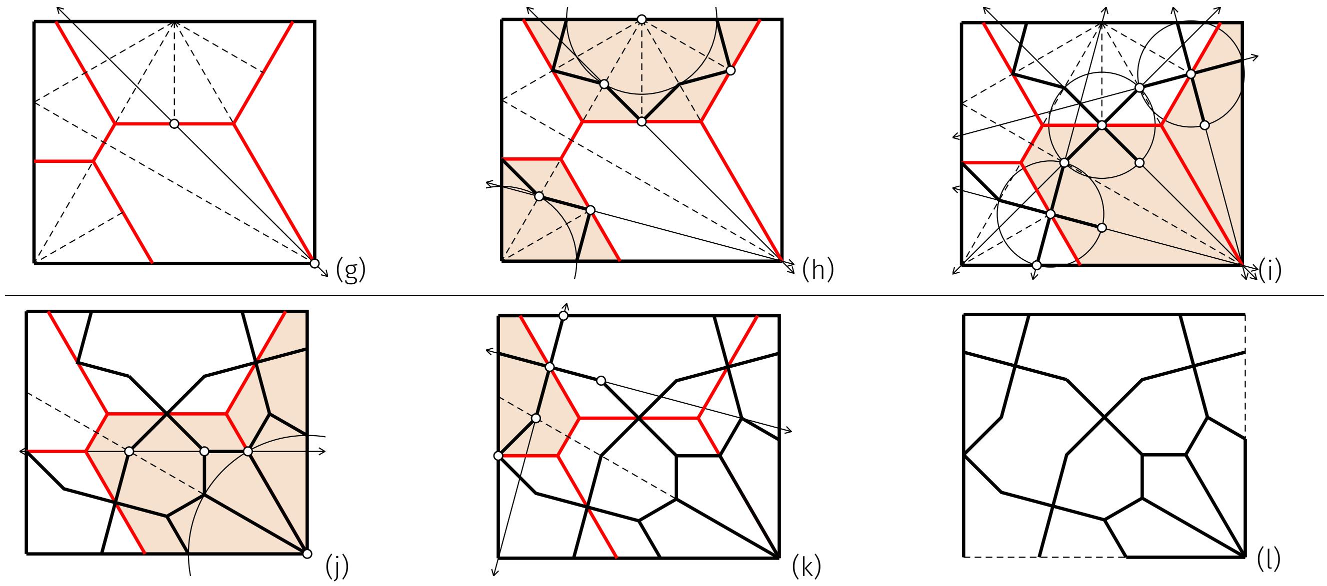


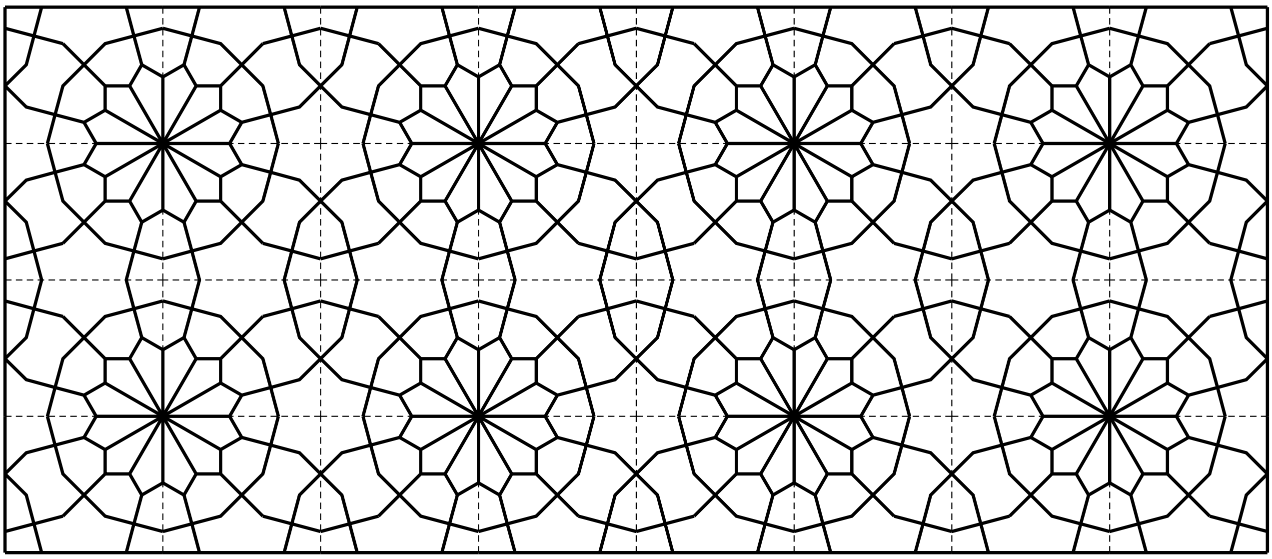
Example 5. Dodecagonal design from Agra fort

This design was created on the network of regular triangles, and one can analyze it by drawing triangles on the photograph. But we can also draw it in a rectangular contour.

This time we will follow the second option. It is more interesting.

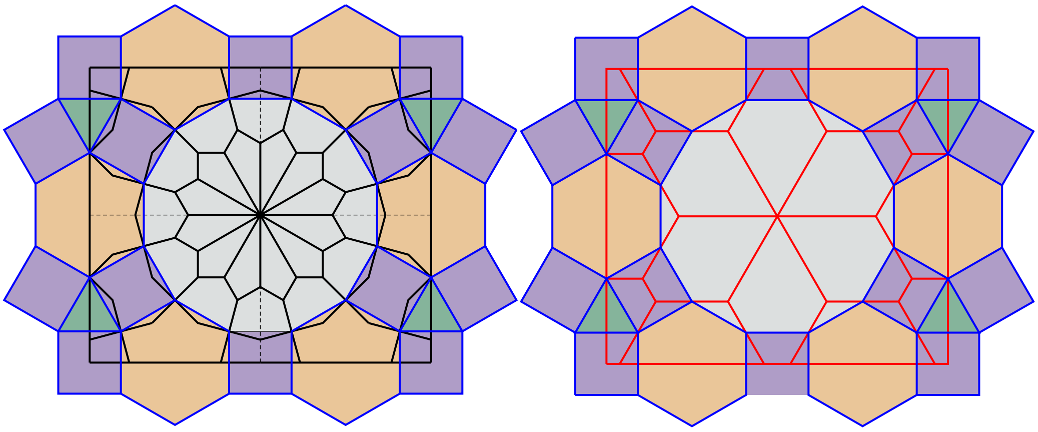






In this series of drawings worth noticing are two pictures (b) and (g). The first one shows the creation of the contour for this design. In drawing (g), we show how the first line for the future pattern was created. The whole pattern is a natural outcome of this line and axioms of gereh (see [2])

It is interesting to look at tessellations for this design. We have already seen one tessellation for this pattern. Do we have such tessellations more? The next drawing shows one of a few other tessellations that can be used to design this pattern. I suggest the reader look for more tessellations for this pattern.



Two tessellations for the dodecagonal pattern from Agra fort

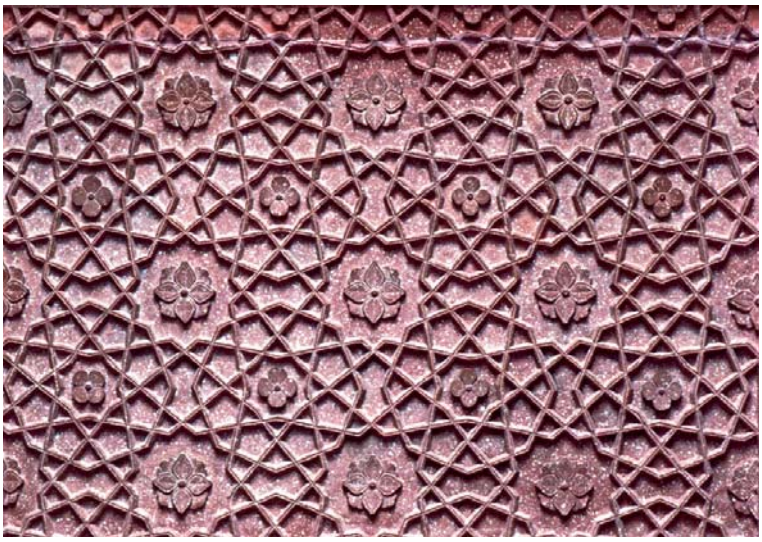
(Left) In blue, the new tessellation is somewhat inconsistent with the gereh rules, but it also has a nice feature – it is a tessellation of regular polygons only.

(Right) The relation between the red tessellation and the blue one.

As we can see, tessellations for patterns with local dodecagonal symmetries often are tessellations of regular polygons. This type of tessellations was investigated by many mathematicians and artists in the past as well as nowadays. Here we can mention Johannes Kepler, M.C. Escher, and a few others.

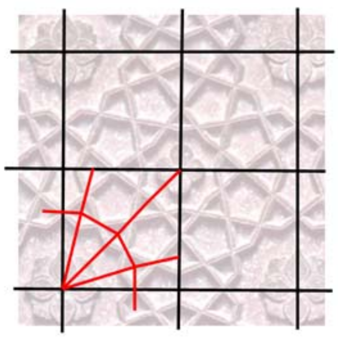
Mixed local symmetries

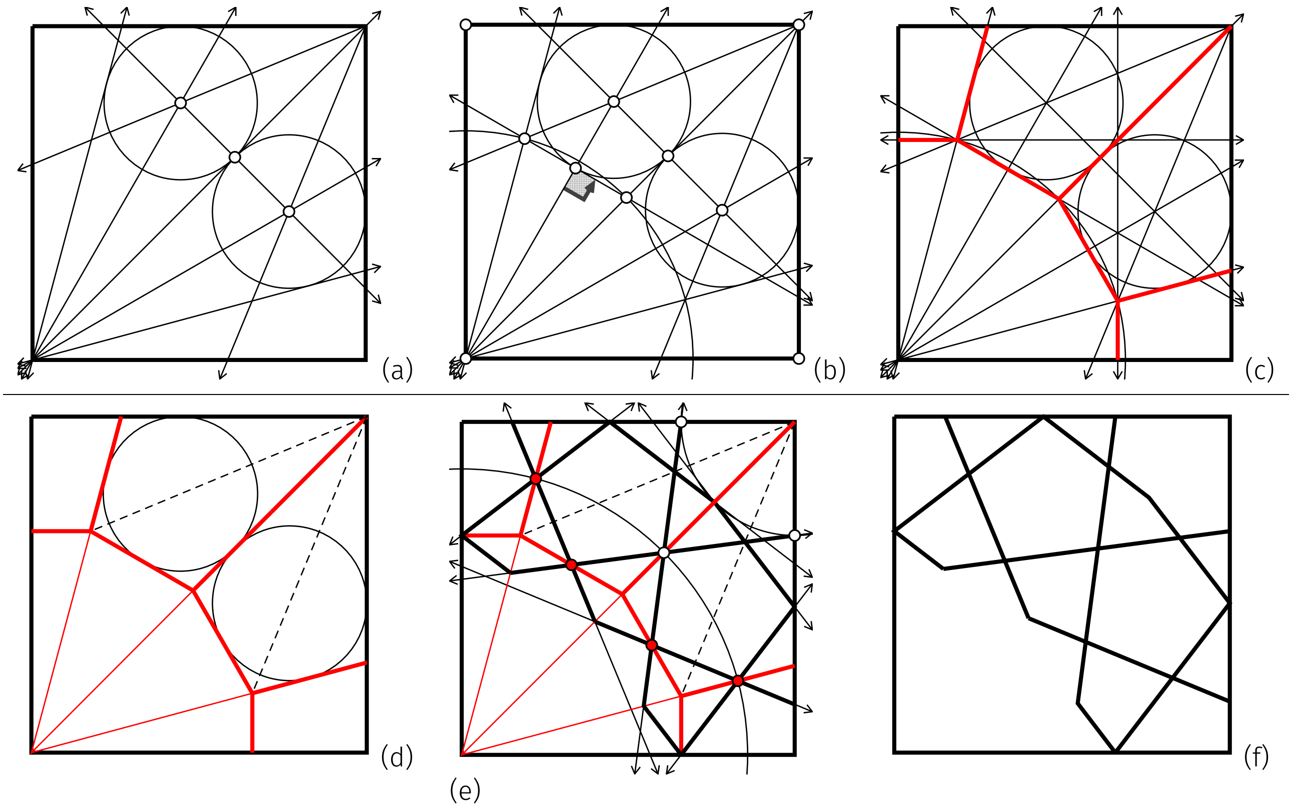
In the previous design, we already had two types of local symmetry – dodecagonal and hexagonal. But hexagonal symmetry is a part of the dodecagonal symmetry (12 is divisible by 6). Such symmetries we will consider as compatible. Here is another example using two types of symmetry D12 and D8. This type of combination of symmetries we call weak-compatible. Number 8 does not divide 12, but they have a common factor, the number 4.

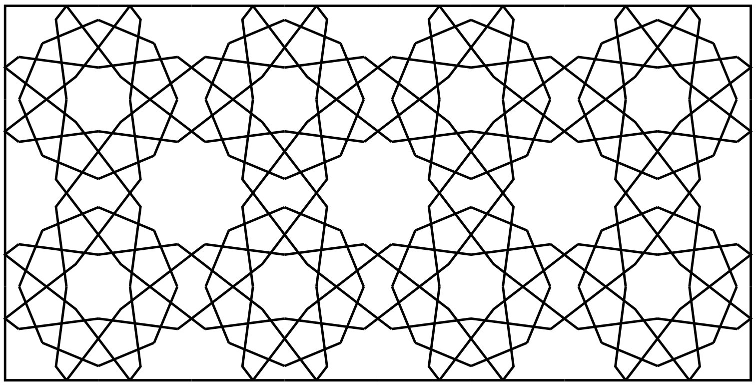


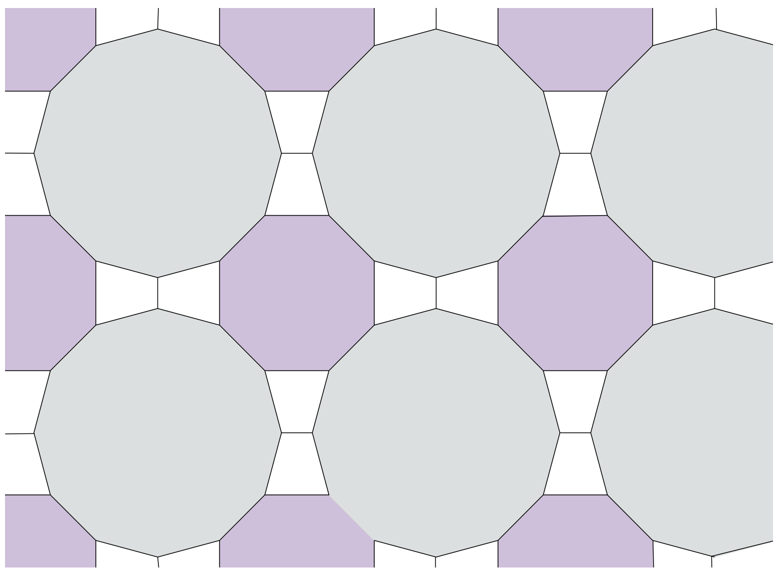
Example 6. Another geometric pattern from Agra fort

In the photo, we have one of many designs from Agra fort. The drawing shows lines of symmetry as well as a possible tessellation for this design.









Reconstruction of the pattern from Agra fort

In drawings (a) to (d), the two circles are used to get a reasonably regular shape of pentagons. This way, we produce tessellation with all tiles symmetric. The red points in (e) are midpoints of the edges of tessellation tiles.

This pattern gives us some freedom in constructing tessellation and the final artwork. For example, one can create a tessellation with regular dodecagons and regular octagons and trapeziums between them (left image) and then fill both regular polygons with stars in such a way that the edges of each star inside the dodecagon will continue without bending inside neighboring octagons. This may result in different angles and slightly different shapes. As a consequence, the pentagons in this pattern may be more or less regular.

Regular and semiregular polygons

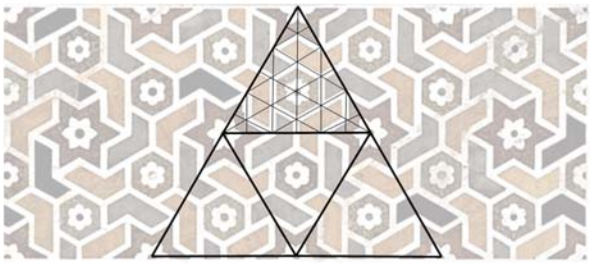
While exploring Mughal designs, we often see geometric patterns that can be created using tessellations of regular polygons. We have seen them in some of the earlier discussed examples. Such designs often use squares, equilateral triangles, regular hexagons, and regular dodecagons. These patterns are relatively rare in Moghul art. But we often see designs created on tessellations of regular octagons and squares. A large number of such designs we can find in Fatehpur Sikri in Rajah Birbal's house. An extensive collection of these patterns was published by Smith E.W. in [6]. However, the quality of photographs in available scans is inferior.

Patterns using triangular and square grids

Numerous geometric designs in Moghuls' architecture were created using the so-called isometric grids and various square grids. We will discuss here only two such patterns, but the collection of them is much more extensive. Thus let us start with a design on an isometric grid.

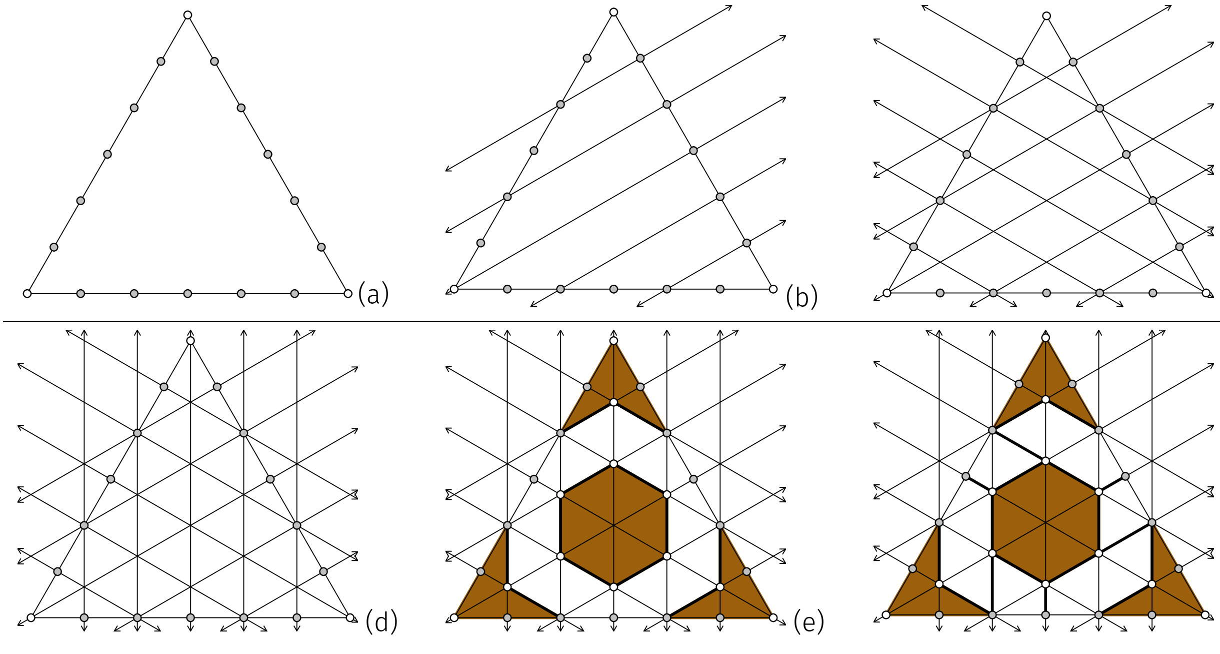
Example 7. Panel from the Tomb of Itimud ad-Daula in Agra

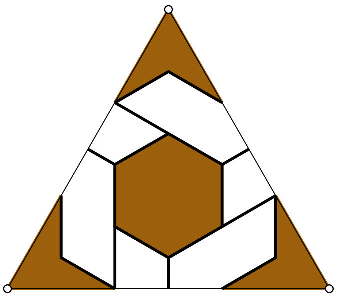
One can easily see that we have a series of segments following vertical or slant lines on this pattern. This design has no mirror symmetry lines. The center of each star can be treated as a center of rotation D6 if we ignore colors. We can say the same about centers of hexagons.



Isometric grid for the pattern from the Itimud ad-Daula in Agra

By drawing segments joining centers of stars, we get large triangles, and each of these triangles can be subdivided into a more dense isometric grid. Note, each edge of the large triangle was divided into 6 equal parts, and then we produce lines perpendicular to it.

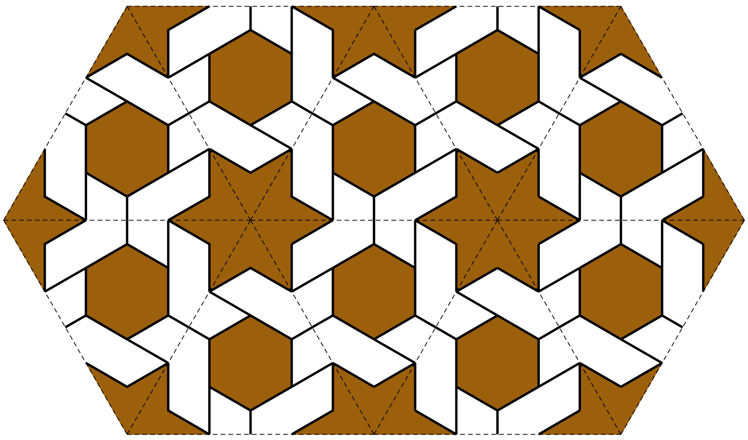




(g)

All steps in the construction of the pattern from the Tomb of Itimud ad-Daula in Agra

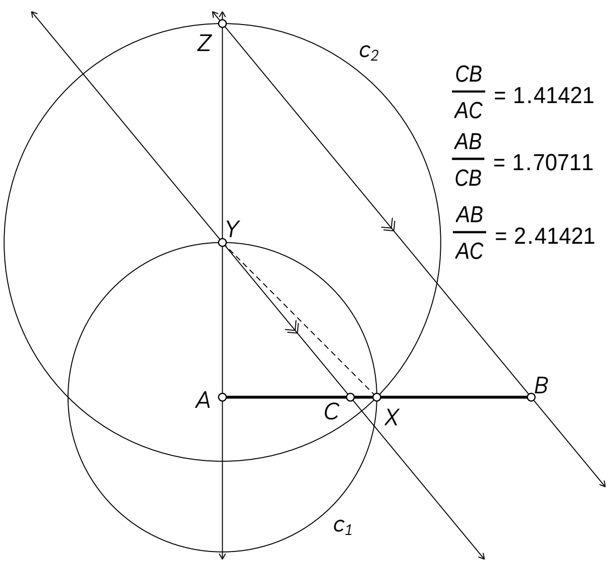
We proceed from a single equilateral triangle by creating the isometric grid, then drawing pattern by following lines of the grid, and finally rotating the obtained triangular template (drawing (g)) 60 degrees around one of its vertices. The next picture shows a larger pattern obtained from (g). The top-left triangle is the original template.



There are numerous geometric decorations in Mughal architecture that can be approached using combinations of particular rectangular grids. These are grids with variable distances between parallel lines, and they can be used to produce several interesting geometric patterns. Some of them we have seen already (example 2). Here is one of many instances where we could apply rectangular grids. We will start with the construction of the side-and-diagonal section or, in mathematical terms a $\sqrt{2}$ section of a segment.

Construction of the side-and-diagonal section

Suppose we have a segment AB and we want to split it into two parts AC, and CB, such that CB/AC=$\sqrt{2}$ or AC/CB= $\sqrt{2}$.



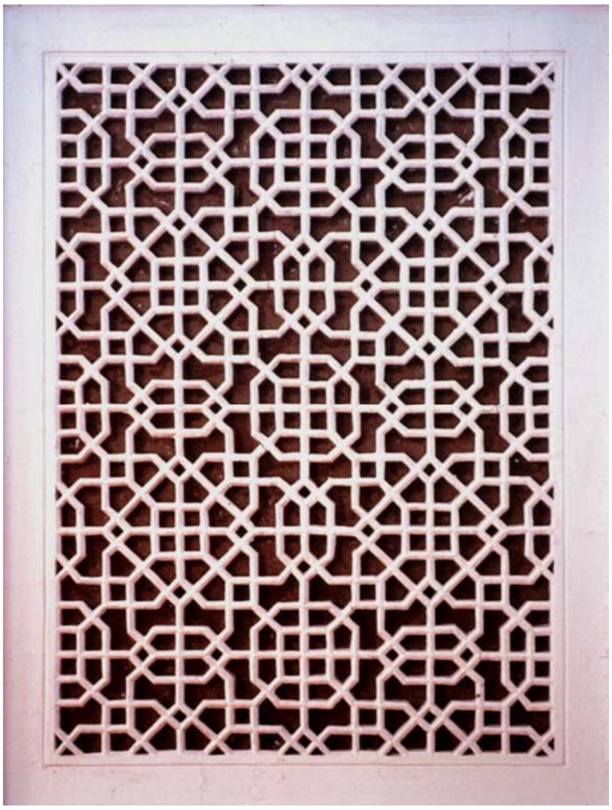
The side-and-diagonal section

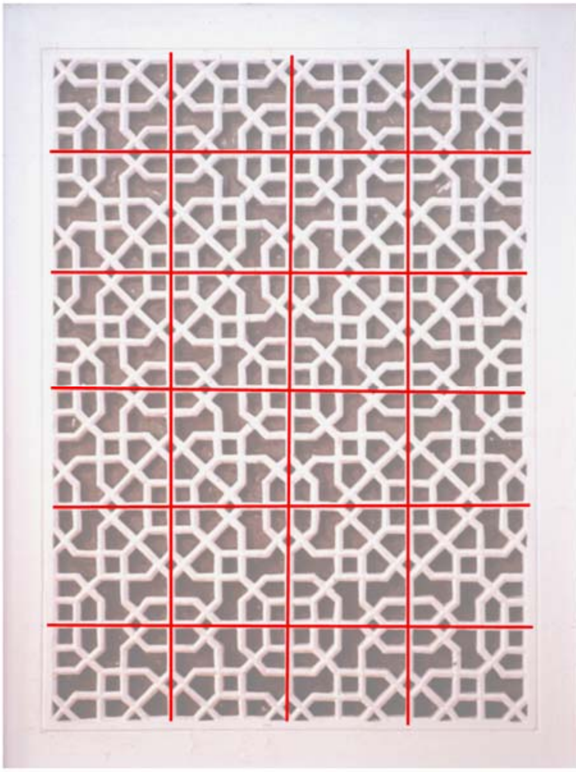
We start by drawing a segment *AB*. Then we choose on it any point *X*. It can be a completely random point. After drawing the circle c_1 we get the point *Y*. The segment *YX* can be treated as a diagonal of the square with side *AX*. By drawing the next circle c_2, we get the point *Z*. Thus, on the vertical line, we have two segments *AY* and *YZ*. It is evident that they fulfill that $ZY/AY=\sqrt{2}$.

By drawing line *ZB* and parallel to it the line passing through point *Y*, we obtain the required point *C*.

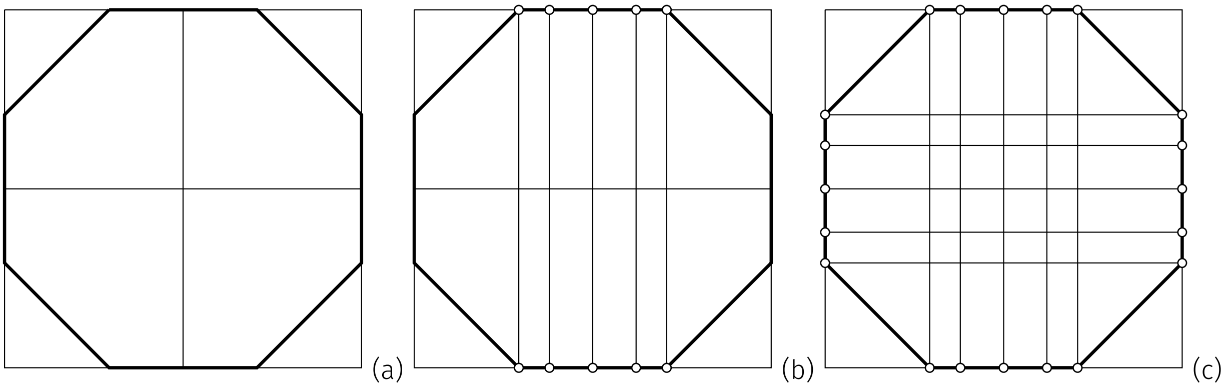
On the drawing are shown two other ratios that often occur in octagonal geometric patterns.

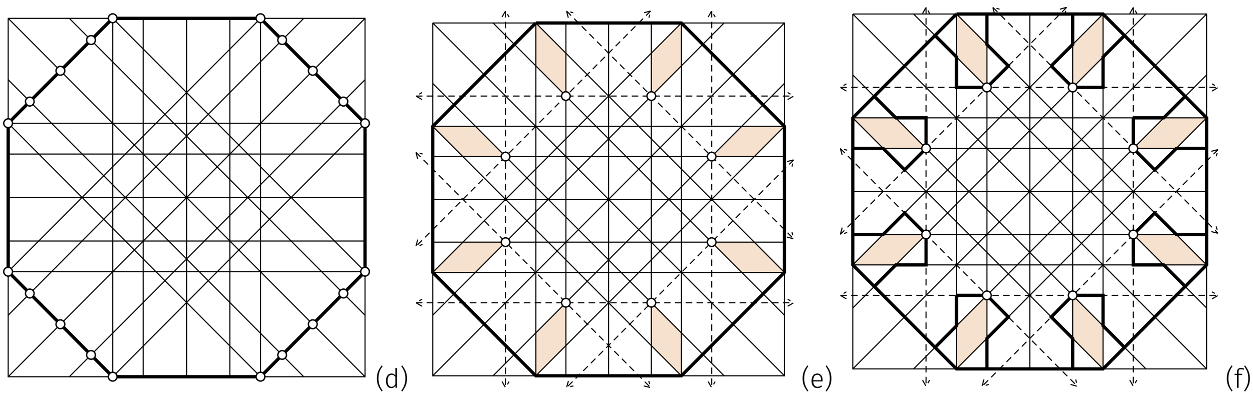
Example 8. Window lattice from the Maharajah's Palace in Jaipur

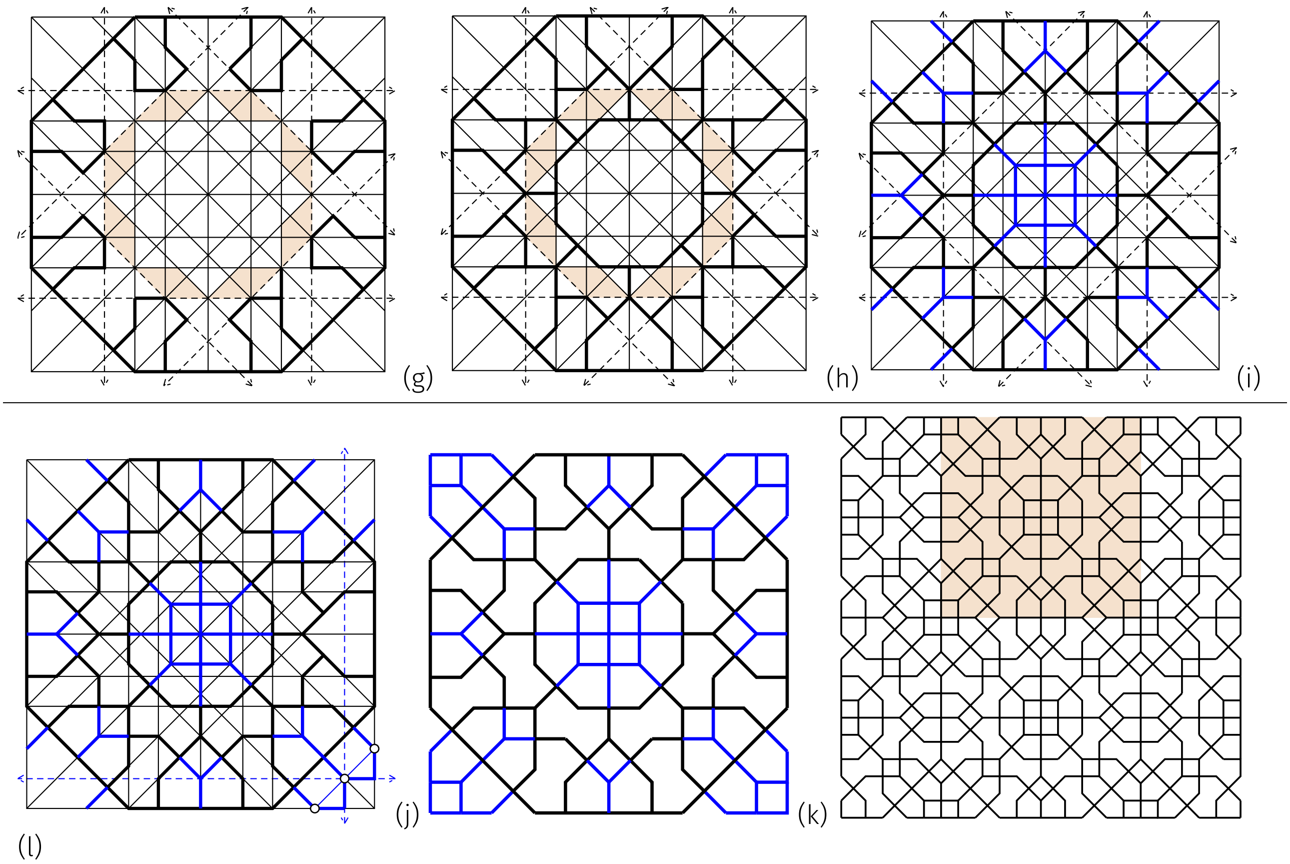




The left photo shows a typical lattice that one can see in Moghul's buildings. It is a maze of segments made out of overlapping and tangent octagons. In the right drawing was see some of the mirror symmetry lines of this pattern. For designing this decoration, we need to construct a pattern in one of these squares. However, it is more convenient to use four such squares together.







Construction of the window lattice from Maharajah's Palace from Jaipur

We start from a square template with an inscribed inside it octagon. Then we split each side of the octagon into two equal parts, and we divide each piece into smaller parts using the side-and-diagonal section. Larger segments of the section are in the middle of each side. We use these divisions to create two overlapping rectangular grids (d).

In the drawing (e), we identify eight narrow rhombi (shaded polygons), and we use vertices of them to add four extra lines to our grids. Now we can start drawing parts of the pattern by wrapping each rhombus in a hexagonal shape (f).

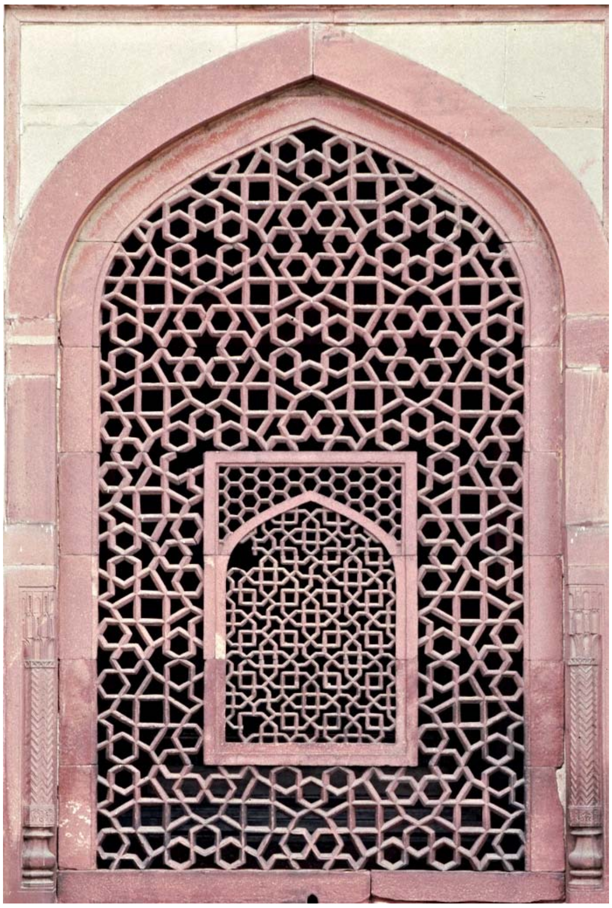
We repeat this process again to the new rhombi obtained in (g).

The remaining steps of this construction are easy to follow from the enclosed images.

The last image shows the top part of the pattern from the Maharajah's Palace. The shaded area is the fragment that we obtained as a result of this construction.

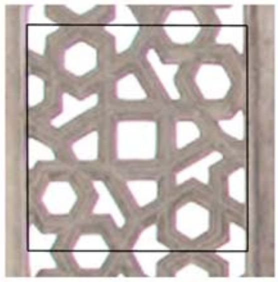
The rectangular grids using side-and-diagonal section we will call variable rectangular grids. The location of wide and narrow bands may change depending on a pattern we deal with. While dealing with octagonal designs from the Mughal Empire, we can find more examples that can be reconstructed using the variable rectangular grids. We can see this technique in a few other geometric patterns in Fatehpur Sikri in Rajah Birbal's house. However, the design discussed in this section is the most complex one. A few more complex geometric patterns suitable for this technique can be seen in Iran and further to the west, in Morocco. Moroccan pattern designers developed a method known there as the hasba method. This method uses quite elaborate grids made out of parallel lines. Patterns created on these grids are one of the most complex.

This paper's last example shows a geometric pattern that does not fit into any of the groups discussed in this paper. This pattern is part of another window lattice. This time it is on the window in the Taj Mahal.

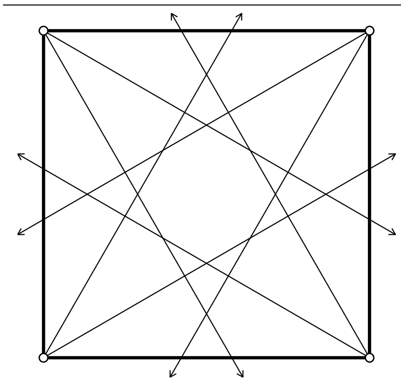


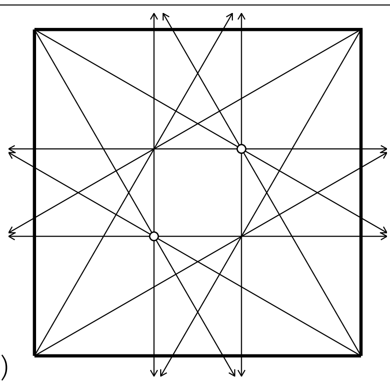
Example 9. The Taj Mahal window lattice

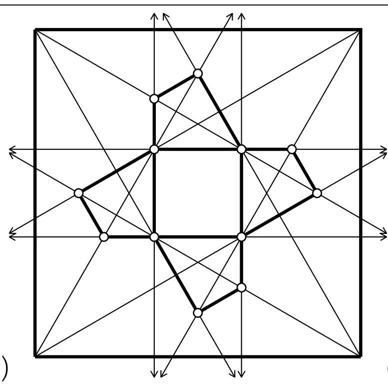
In the photograph, we have a combination of two lattices, and thus we have two different geometric patterns. The external lattice will be our target. From the photograph, we can easily find out that we need to reconstruct the square area between four hexagonal stars with stars' centers as vertices of the square. This is shown in the image enclosed below.

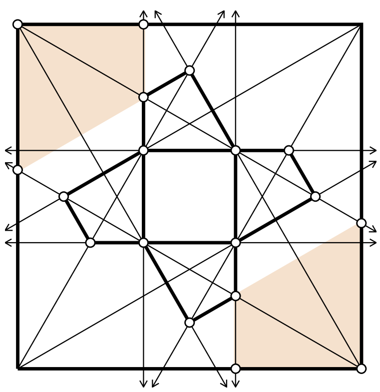


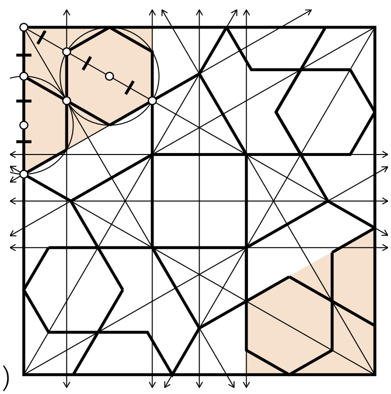
The reconstruction of this design will be a combination of various grids. We show how it can be done on the following drawings. This design has a few other ways of looking at it and reconstructing it. The small template of this design is shown in illustration (e). The large template, next drawing, was produced by using mirror reflections of the small template.

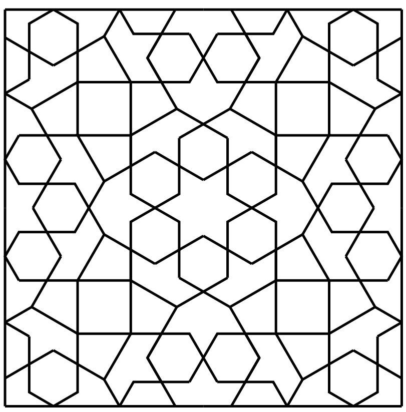

(a)


(b)


(c)


(d)


(e)



Summary

In this paper, we explored various types of geometry used in geometric patterns in Mughal architecture. Each type of geometry uses different angles and different shapes of the elements seen in the pattern. Although most of the geometric designs from Mughal architecture are very simple, we still have a very diverse collection of them. One can notice that in this collection, octagonal designs form the largest group and the use of square and octagon is the most frequent. The group of decagonal designs is very limited, and we do not see in India such complexity as in decagonal patterns in Iran and further to the west. However, even in this group, one can find some excellent works. The very neglected ceramic decoration from the wall of the Shamsuddin Muhammad Atgah Khan (Ataga Khan), (died May 15, 1562) tomb in Delhi is an excellent example of a simple triangular structural design.

In Mughal patterns, we often see simple geometric constructions often using regular polygons – equilateral triangle, square, pentagon, octagon, decagon, and dodecagon. We do not have some advanced pattern designing techniques like various types of polygonal inflation used by masters in Iran or structural designs like those in Ottoman geometric designs. Thus geometric patterns from India are an excellent material to be investigated in high school geometry classes.

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